PARTICLE SPREADING DURING ACCELERATION AND PENETRATION

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On the basis of models of viscous incompressible and ideal fluids as well as an elastic medium, the destruction of spheroidal particles under the unilateral effect of high pressure corresponding to acceleration or penetration conditions is considered herein. The pressure and velocity fields here are found, the characteristic angles of spreading and the size of the nondestructive compact zones are determined, estimates are given of the specific impulse resulting in complete spreading of the particles, etc.

A theoretical solution is obtained in Legendre polynomials by separation of variables by using the Laplace transform (for the nonstationary problem). The experiments conducted herein agree with theory.

Investigations on the destruction of solid particles during collisions with obstacles and during acceleration were started at the beginning of the nineteenth century [1] and are intensively conducted at present [2-5].

The destruction of spheroidal particles under the unilateral effect of high pressure corresponding to acceleration or penetration conditions is examined herein on the basis of models of a viscous incompressible and ideal fluid and also an elastic medium. These question are of interest for the punch-through problem since the interaction parameters [5] as well as the maximum velocity which can be obtained during particle acceleration depend on the degree of particle destruction. The experiments conducted agree with theory.

1. Quasistationary Spreading of Spherical Particles. Let us examine the problem of slow quasistationary spreading of a viscous spherical particle of diameter $d_0 = 2r_0$ subjected to a unilateral pressure (Fig. 1) corresponding to acceleration or penetration.

Let us consider the particle material to behave as a viscous incompressible fluid under conditions of high compressions on the order of 10^{11} bar. This approach has been developed in [2] in describing the process of cumulative jet penetration in metals by the hydrodynamics equations of an incompressible fluid. At such pressures the metal compressibility is several percent and the viscous stresses $\mu_0 \partial V_z/\partial z$ ($\mu_0 = 10^5$ P) exceed the yield point σ_T [6-9] although the Reynolds number remains low ($Re = \rho_0 U r_0/\mu_0 < 1$).

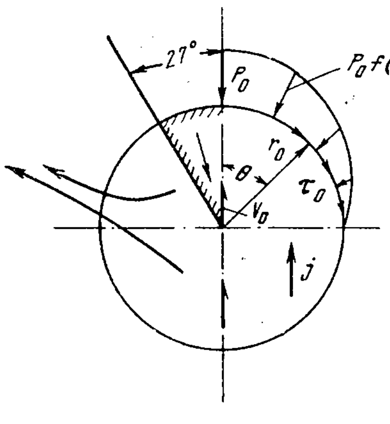

Fig. 1

For a quasistatic analysis it is necessary that the deceleration (acceleration) time t_0 be greater than the characteristic wave time $t_W = d_0/c_0$ and the pressure should vary smoothly on the particle, without causing spalling.

Neglecting quadratic inertia terms and dV/dt in the hydrodynamics equations, let us write them as

$$\operatorname{grad} p = \rho \mathbf{j} + \mu_0 \Delta \mathbf{V}, \quad \operatorname{div} \mathbf{V} = 0, \quad \mathbf{j} = -F/m_0$$
$$m_0 = {}^4/_3 \pi r_0^3 \rho_0 \tag{1.1}$$

where F is the principal vector of the surface forces.

Let us direct the inertial force vector j along the z axis opposite to the resultant of all the surface forces in a moving spherical coordinate system coupled to the center of the particle.

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Applying the operation div $\mathbf{V} = 0$ to (1.1), we obtain the Laplace equation $\Delta p = 0$ for the hydrostatic pressure.

The conditions of equality of stresses on the ball surface

$$\sigma_{rr}(r_0, \theta) = -p + 2\mu_0 \partial V_r/\partial r = -P_0 f(\theta)$$
$$\sigma_{r\theta}(r_0, \theta) = \mu_0 (1/r \partial V_r/\partial\theta + \partial V_\theta/\partial r - V_\theta/r) = \tau_0 \Phi(\theta) \tag{1.2}$$

are the boundary conditions for (1.1).

The solutions should satisfy the continuity equation since the order of the system is raised by one after the operation div has been applied to (1.1) to derive the equation $\Delta p = 0$. Let us seek the solution by separation of variables:

$$V_r = \sum_{n=0}^{\infty} \psi(r) P_n(\cos\theta), \quad V_\theta = \sum_{n=0}^{\infty} \varphi(r) \partial P_n(\cos\theta)/\partial\theta$$
$$p = \sum_{n=0}^{\infty} N_n r^n P_n(\cos\theta) \tag{1.3}$$

We obtain the system of equations

$$r^{1-n}\psi_n'' + 2r^{-n}\psi_n' - [n(n+1)+2] r^{-n-1}\psi_n + 2n(n+1) r^{-n-1}\psi_n = nN_n/\mu_0$$
$$r^{1-n}\varphi_n'' + 2r^{-n}\varphi_n' - n(n+1) r^{-n-1}\varphi_n + 2r^{-n-1}\psi_n = N_n/\mu_0$$

for the functions $\psi(r)$ and $\varphi(r)$, and from their solution we determine the desired functions and find expressions for the velocity and pressure fields:

$$V_r = B_1\left[1 - \left(\frac{r}{r_0}\right)^2\right]\cos\theta + \sum_{n=2}^{\infty} r_0^{n+2}\left[A_n\left(\frac{r}{r_0}\right)^{n+1} + B_n\left(\frac{r}{r_0}\right)^{n-1} r_0^{-2}\right] P_n(\cos\theta)$$
$$V_\theta = -B_1\left[1 - 2\left(\frac{r}{r_0}\right)^2\right]\sin\theta + \sum_{n=2}^{\infty} r_0^{n+2}\left[\frac{n+3}{n(n+1)} A_n\left(\frac{r}{r_0}\right)^{n+1} + \frac{1}{n} B_n\left(\frac{r}{r_0}\right)^{n-1} r_0^{-2}\right]\frac{\partial P_n(\cos\theta)}{\partial\theta} \tag{1.4}$$
$$p = A_0 + (10\mu_0 A_1 + \rho j) r\cos\theta + \sum_{n=2}^{\infty} \frac{2\mu_0(2n+3)}{n} A_n r^n P_n(\cos\theta)$$

To determine the constants A_n and B_n, we expand the external pressure $P_0 f(\theta)$ in a series of Legendre polynomials and the friction $\tau_0\Phi(\theta)$ in associated polynomials in the interval $(0, \pi)$:

$$f(\theta) = \sum_{n=0}^{\infty} f_n P_n(\cos\theta), \quad \Phi(\theta) = \sum_{n=1}^{\infty} \Phi_n \frac{\partial P_n(\cos\theta)}{\partial\theta}$$
$$f_n = \frac{2n+1}{2}\int_0^{\pi/2} f(\theta)\sin\theta P_n(\cos\theta)\,d\theta \tag{1.5}$$
$$\Phi_n = \frac{2n+1}{2}\frac{(n-1)!}{(n+1)!}\int_0^{\pi/2} \Phi(\theta)\sin\theta\,\partial P_n(\cos\theta)/\partial\theta$$

Upon penetration of particles in a dense medium the surface friction forces are proportional to the normal pressure $\tau = kP(r_0\theta)$. For metals k is on the order of several percent so that $\tau \ll P$ in this case. Let us examine the case, often encountered, in which $\tau = 0$ and the external pressure is expressed by the Newton formula $P(r_0, \theta) = \rho_1 V_0^2 f(\theta)$, where

$$f(\theta) = \begin{cases}\cos^2\theta, & 0 \leqslant \theta \leqslant \pi/2 \\ 0, & \pi/2 < \theta \leqslant \pi\end{cases} \tag{1.6}$$

The solution for the velocities and the hydrostatic pressure becomes

$$V_r = \frac{P_0 r_0}{\mu_0}\sum_{n=2}^{\infty}\frac{n f_n}{2[2n(n+2)+3]}\left[(n+1)\left(\frac{r}{r_0}\right)^{n+1} - \frac{n(n+2)}{n-1}\left(\frac{r}{r_0}\right)^{n-1}\right] P_n(\cos\theta)$$
$$V_\theta = \frac{P_0 r_0}{\mu_0}\sum_{n=2}^{\infty}\frac{f_n}{2[2n(n+2)+3]}\left[(n+3)\left(\frac{r}{r_0}\right)^{n+1} - \frac{n(n+2)}{n-1}\left(\frac{r}{r_0}\right)^{n-1}\right]\frac{\partial P_n(\cos\theta)}{\partial\theta} \tag{1.7}$$

$$p = f_0 P_0 + f_1 P_0 \frac{r}{r_0} \cos\theta + P_0 \sum_{n=2}^{\infty} \frac{(n+1)(2n+3)}{2n(n-2)-3} f_n \left(\frac{r}{r_0}\right)^n P_n(\cos\theta)$$

It follows from (1.7) that the spreading velocities diminish in the interior domains, reaching zero values at the center, although the velocity gradients increase as $r \to 0$. Outward spreading of its material is observed in the majority of particle domains. But a so-called compact head zone with inward motion of the material exists in the particle from the side of the applied pressure. This domain is in the forward sector with a central angle of 54° (Fig. 1).

The particle is deformed during spreading.

It can be considered that the particle is destroyed completely when the radial displacements

$$U_r = \int_0^t V_r dt$$

merge at $\theta = 0$ and π. The expression to determine these total displacements is found from (1.7) and is

$$\Delta = \frac{(|U_r(r_0, 0)| + |U_r(r_0, \pi)|)}{r_0} = 0.18 I / \mu_0,$$

$$I = \int_0^t P(r_0, 0, t)\, dt \tag{1.8}$$

For $\Delta = 2$ we find the critical value of the specific impulse $I_c = 12\mu_0$. Experiments show that metal particles are destroyed at the radial deformation $\Delta = 1$, to which value the critical impulse $I_c = 6\mu_0$ corresponds from (1.8).

The maximum velocity of the thrown (or decelerating during penetration) particle can be expressed in terms of I_c if the interaction conditions $p_0 > \sigma_T$ result in viscous flow:

$$V_c = \frac{f_1 I_c}{\rho_0 r_0} = \frac{\mu_0}{\rho_0^{2/3}} \left(\frac{4\pi}{3m_0}\right)^{1/3} \xi, \quad \xi = 2.5 - 5, \quad f_1 = \frac{3}{8}$$

The particle is heated strongly because of internal friction during viscous spreading.

Let us consider the deceleration (acceleration) time τ_0 to be much less than the characteristic heating time. Then the dissipated energy in Lagrange coordinates is conserved.

The Eulerian and Lagrangian coordinates coincide as $r \to 0$ in a coordinate system where the center is fixed, and the terms of the expansion (1.7) yield a contribution to the temperature only for $n = 2$, i.e.,

$$\rho_0 C_p \frac{dT}{dt}\bigg|_{r \to 0} = \frac{192}{361} (f_2 P_0)^2 \mu_0(T), \quad f_2 = \frac{1}{3} \tag{1.9}$$

The initial heating drops because of heat conduction and the temperature dependence of the viscosity so that (1.9) yields the upper bound of the temperature.

Not all the particle material goes over into the viscoplastic state under unilateral loading. Hence, attempts to obtain an accurate picture of the stress state in the particle are fraught with great difficulties since solutions satisfying the viscoplastic and elastic states of the material must be joined on an unknown boundary. The stress state can be investigated qualitatively in the example of the exact solution of the elastic problem, which is also of independent interest since brittle materials, for example, are destroyed at low deformations subject to Hooke's law.

General methods of solving the elasticity theory equations are known [10]. However, the case of a unilateral load on the sphere according to the Newton formula is of special interest since it corresponds to the quasistationary penetration of solid particles or to their acceleration in a plasma stream. This problem has been examined independently in [13] and by the authors. Hence, let us limit ourselves just to the main deductions of the theory.

Setting $\sigma_{rr}(r_0, \theta) = -P(r_0, \theta)$ and giving the boundary conditions in the form (1.6), applying separation of variables to the elasticity theory equations in displacements, we obtain the desired solutions for the stress tensor components:

$$\sigma_{rr} = -f_0P_0 - f_1P_0\left(\frac{r}{r_0}\right)\cos\theta + \sum_{n=2}^{\infty} H\left\{[n(n-1) - 2(1+\eta)]\left(\frac{r}{r_0}\right)^n - \frac{n(n+1)^2 - 2n(1-\eta)}{n+1}\left(\frac{r}{r_0}\right)^{n-2}\right\} P_n(\cos\theta)$$

$$\sigma_{\theta\theta} = -f_0P_0 - f_1P_0\left(\frac{r}{r_0}\right)\cos\theta + \sum_{n=2}^{\infty} H\left\{[n(1-4\eta) - 2(1+\eta)] \times \right.$$

$$\left. \times \left(\frac{r}{r_0}\right)^n - \frac{n(n+1) - 2n(1-\eta)}{n^2-1}\left(\frac{r}{r_0}\right)^{n-2}\right\} P_n(\cos\theta) +$$

$$+ H\left\{\frac{n+5-4\eta}{n+1}\left(\frac{r}{r_0}\right)^n - \frac{(n+1) - 2(1-\eta)}{n^2-1}\left(\frac{r}{r_0}\right)^{n-2}\right\} \times \frac{\partial^2 P_n(\cos\theta)}{\partial\theta^2}$$

$$\sigma_{\varphi\varphi} = -f_0P_0 - f_1P_0\left(\frac{r}{r_0}\right)\cos\theta + \sum_{n=2}^{\infty} H\left\{[n(1-4\eta) - 2(1+\eta)] \times \right.$$

$$\left. \times \left(\frac{r}{r_0}\right)^n - \frac{n(n+1) + 2n(1-\eta)}{n^2-1}\left(\frac{r}{r_0}\right)^{n-2}\right\} P_n(\cos\theta) +$$

$$+ H\left\{\frac{n+5-4\eta}{n+1}\left(\frac{r}{r_0}\right)^n - \frac{(n+1) - 2(1-\eta)}{n^2-1}\left(\frac{r}{r_0}\right)^{n-2}\right\} \frac{\partial P_n(\cos\theta)}{\partial\theta}\operatorname{ctg}\theta$$

$$\sigma_{r\theta} = \left[1 - \left(\frac{r}{r_0}\right)^2\right] \sum_{n=2}^{\infty} H \frac{(n+1)^2 - 2(1-\eta)}{n+1}\left(\frac{r}{r_0}\right)^n \frac{\partial P_n(\cos\theta)}{\partial\theta}$$

$$H = \frac{(n+1)f_nP_0}{2n^2 + 2n(1+2\eta) + 2(1+\eta)}$$

where η is the Poisson ratio.

It is seen from an analysis of the stress state that it varies along the particle volume and has a singularity in each zone. The maximum tangential stresses which originate initially at the center and are propagated towards the circumference to result in shear strains as P_0 grows are characteristic for the zone $\theta = \pi/4$. Along the radius, $\theta = \pi \sigma_{\theta\theta} = 0$ and $\sigma_{\varphi\varphi} = 0$; hence, one-dimensional compression of the material due to the effect of inertial forces is observed in this domain.

The zone $\theta = \pi/2$ contains the tensile stresses $\sigma_{\varphi\varphi}$ which are governing during destruction of brittle particles.

Multilateral compression of the material holds in the neighborhood of the critical point $\theta = 0$. This domain is "most stable."

2. Nonstationary Spreading of Spheroidal Particles. The solutions obtained in Sec. 1 can be considered valid on the expiration of the time $t > t_0$ when the product of the Strouhal and Reynolds numbers becomes much less than one. It is interesting to study the nature of the emergence of the flow into this quasistationary mode under the assumption of smallness of the change in the particle outline during the time t_0 and of smallness of Re.

The solution of the problem is valid for nonstationary boundary conditions when the pressure applied to the particle depends on the time. Since $\mathrm{Re} \ll 1$ and $t_0 \gg r_0/c$, the equations of motion in the moving coordinate system are

$$\rho_0 \partial \mathbf{V}/\partial t + \operatorname{grad} p = \mu_0 \Delta \mathbf{V} + \rho j, \quad \operatorname{div} \mathbf{V} = 0, \quad \mathbf{j} = \mathbf{F}/m_0 \tag{2.1}$$

with the boundary and initial conditions

$$\sigma_{rr}(r_0, \theta, t) = -P_0(t) f(\theta), \quad \sigma_{r\theta}(r_0, \theta, t) = 0$$
$$V_r(r, \theta, 0) = 0 \qquad V_\theta(r, \theta, 0) = 0 \tag{2.2}$$

Applying to (2.1) and (2.2) the Laplace transform

$$U_r \risingdotseq V_r, \quad \Pi_0 \risingdotseq P_0(t), \quad \Pi \risingdotseq p,$$

we obtain

$$\rho_0 S\mathbf{U} + \operatorname{grad}\Pi = \mu_0 \Delta \mathbf{U} + \rho \mathbf{j}, \quad \operatorname{div}\mathbf{U} = 0, \quad \Delta\Pi = 0 \tag{2.3}$$

$$-\Pi(r_0, \theta) + 2\mu_0 \left.\frac{\partial U_r}{\partial r}\right|_{r=r_0} = -\Pi_0(S) f(\theta)$$

$$f(\theta) = \sum_{n=0}^{\infty} f_n P_n(\cos\theta) \tag{2.4}$$

$$\left(\frac{1}{r}\frac{\partial U_r}{\partial\theta} + \frac{\partial U_\theta}{\partial r} - \left.\frac{U_\theta}{r}\right|_{r=r_0} = 0, \quad u_r(r, \theta, 0) = 0 \quad u_\theta(r, \theta, 0) = 0\right)$$

The variables in (2.3) separate into Legendre polynomials:

$$\Pi = K_0 + K_1^0 r \cos\theta + \sum_{n=2}^{\infty} K_n r^n P_n(\cos\theta)$$

$$u_r = \sum_{n=0}^{\infty} \psi(r) P_n(\cos\theta) \tag{2.5}$$

$$u_\theta = \sum_{n=1}^{\infty} \varphi_n(r) \frac{\partial P_n(\cos\theta)}{\partial\theta}, \quad \varphi_n = \frac{1}{n(n+1)}\left(r\frac{\partial\psi_n}{\partial r} + 2\psi_n\right)$$

Substituting (2.5) into (2.3), we obtain equations to determine ψ_n and φ_n. The solutions ψ_n^0 and φ_n^0 of the corresponding homogeneous equations are

$$\psi_0^0 = A_0 x^{-1/2} I_{3/2}(x), \quad \psi_n^0 = A_n x^{-1/2} I_\alpha(x) + B_n x^{-1/2} I_\beta(x)$$

$$\varphi_n^0 = C_n x^{-1/2} I_\alpha(x) + D_n x^{-1/2} I_\beta(x), \quad x = r\sqrt{\rho_0 S/\mu_0}$$

$$C_n = -\frac{A_n}{n+1}, \quad C_n = \frac{B_n}{n}, \quad \alpha = n + \frac{3}{2}, \quad \beta = n - \frac{1}{2}$$

$$n \geqslant 1$$

Let us seek particular solutions of the inhomogeneous equations in the form $\psi_n^* = n\varphi_n^*$.

For φ_n^* we obtain the equation

$$x^2 \frac{d^2\varphi_n^*}{dx^2} + 2x\frac{d\varphi_n^*}{dx} - [x^2 + n(n-1)]\varphi_n^* = \frac{K_n}{\mu_0}\left(\frac{\rho_0 S}{\mu_0}\right)^{-(n+1)/2} x^{n+1}$$

for which the set of bounded solutions has the form

$$\varphi_n^* = R_n x^{-1/2} I_\beta(x) - \frac{K_n}{\mu_0}\left(\frac{\rho_0 S}{\mu_0}\right)^{-(n+1)/2} x^{n-1} \tag{2.6}$$

Let us select the constant

$$R_n = 2^\beta \Gamma(\beta+1) \frac{K_n}{\mu_0}\left(\frac{\rho_0 S}{\mu_0}\right)^{-(n+1)/2}$$

such that as $x \to 0$ the principal terms proportional to x^{n-1} are cancelled in the expansion of φ_n^* and the particular solutions (2.6) go over into particular solutions of (1.4).

The constants A_n, B_n and K_n are determined by substituting the solutions $\psi_n = \psi_n^0 + \psi_n^*$ and $\varphi_n = \varphi_n^0 + \varphi_n^*$ into the boundary conditions (2.4), from which we find after manipulations using recursion relations for the Bessel functions

$$A_n = -\frac{2n(n^2-1)\Pi_0 f_n r_0 x_0^{1/2}}{(2n+1)\mu_0 q_n}$$

$$B_n = \frac{2n(n^2-1)\Pi_0 f_n r_0 x_0^{1/2}}{(2n+1)\mu_0 q_n}\left[1 - \frac{2^\beta \Gamma(\beta+1)\theta_n}{2(n^2-1)x_0^\beta}\right]$$

$$\theta_n = 2(n^2-1)I_\beta + \frac{2n(n+2)+3}{2n+3} x_0 I_{\beta+1}(x_0) + \frac{2n(n+2)}{2n+3} x_0 I_{\beta+3}(x_0)$$

$$K_n = \frac{\Pi_0 f_n x_0^2 \theta_n}{(2n+1) q_n r_0^n}$$

$$q_n = \{2(n-1)[2n(n+2)+3] + x_0^2(2n+1)\} I_\alpha(x_0) + [2(2n+1)(n-1) + x_0^2] x_0 I_{\alpha+1}(x_0)$$

$$n \geqslant 2$$

$$A_0 = A_1 = B_1 = K_1 = 0, \qquad K_0 = \Pi_0 f_0$$

The solutions for the transforms are

$$\Pi = f_0 \Pi_0 + f_1 \Pi_0 \frac{r}{r_0}\cos\theta + \sum_{n=2}^{\infty} K_n r^n P_n(\cos\theta)$$

$$U_r = \sum_{n=2}^{\infty} A_n x^{-1/2} I_\alpha(x) + B_n x^{-1/2} I_\beta(x) + n K_n F(x) \tag{2.7}$$

$$U_\theta = \sum_{n=2}^{\infty} - A_n / (n+1) x^{-1/2} I_\alpha(x) + (B_n / n) x^{-1/2} I_\beta(x) + K_n F(x)$$

$$F(x) = \frac{1}{\mu_0} \left(\frac{\mu_0}{\rho_0 S}\right)^{(n+1)/2} [2^\beta \Gamma(\beta + 1) x^{-1/2} I_{n-1/2}(x) - x^{n-1}]$$

Using power series expansions of the Bessel functions and keeping only the principal terms, we can verify that the expressions (2.6) go over into the solution (1.7) for the quasistationary problem as $S \to 0$. The expression $q_n(x)$ for $n \geq 2$ and $x > 0$ contains only positive terms, and $q_n(-x) = (-1)^\alpha q_n(x)$ for $x < 0$. Hence, the equation $q_n(x) = 0$ has no real roots for $x < 0$. It is convenient to seek the pure imaginary roots $q_n(i\lambda_k)$ by substituting $I_\alpha(x) = i^{-\alpha} I_\alpha(\lambda)$, $\lambda = ix$, i.e., to determine them from the expression

$$\{2(n-1)[2n(n+2)+3] - \lambda_{kn}^2(2n+1)\} I_\alpha(\lambda_{kn}) - \lambda_{kn}[2(2n+1)(n-1) - \lambda_{kn}^2] I_{\alpha+1}(\lambda_{kn}) \qquad (2.8)$$

The roots λ_{kn} may not be complex. The Bessel functions I_α and $I_{\alpha+1}$ are represented as power series with real coefficients; hence, if complex roots λ and $\bar{\lambda}$ were to exist in (2.8), then they would be pairwise conjugate. Using (2.8) and the recursion relation $\lambda I_{\alpha+1} = \alpha I_\alpha - \lambda dI_\alpha / d\lambda$ in the evaluation of the integral

$$0 < \int_0^1 \xi I_\alpha(\lambda\xi) I_\alpha(\bar{\lambda}\xi)\, d\xi = \frac{|I_\alpha(\lambda)|^2 [3 - (n-1)(4n^2-1)]}{|2(n-1)(2n+1) - \lambda^2|^2} \qquad (2.9)$$

we arrive at a contradiction since the expression in the right side of (2.9) is negative for $n \geq 2$ and the integral is always positive. This contradiction proves the absence of complex roots in (2.8). The number of real roots in (2.8) is infinite for each n. The velocity of emergence into the stationary mode is determined by the roots closest to zero. Computations by means of tables [11] show that the roots closest to zero for $n = 2$ are $\lambda_{21} = 5.5$ and $\lambda_{22} = 8.8$ and the correction to the stationary mode damps out rapidly in proportion to

$$\exp[-(\mu_0 t / \rho_0 r_0^2)(5.5)^2]$$

Going from the transforms to the original in (2.7), we obtain the exact solution of a nonstationary problem with a fixed contour:

$$p = p_c + \sum_{n=2}^{\infty} \sum_{K=1}^{\infty} C_k \exp\left(-\frac{\mu_0}{\rho_0} \frac{\lambda_{nk}^2}{r_0^2} t\right) \left(\frac{r}{r_0}\right)^n P_n(\cos\theta)$$

$$C_k = \frac{f_n \Pi_0 x_{0k}^2 \theta_n(x_{0k})}{(2n+1) q_n'}, \quad q_n' = \left.\frac{dq_n(x_0)}{dS}\right|_{S=S_k}$$

$$V_r = V_{rc} + \sum_{n=2}^{\infty} \sum_{k=1}^{\infty} [A_n(x_{0k}) x_k^{-1/2} I_\alpha(x_k) + B_n(x_{0k}) x_k^{-1/2} I_\beta(x_k) + nK_n(x_{0k}) F(x_k)] \frac{q_n(x_{0k})}{q_n'} \exp\left[-\frac{\mu_0}{\rho_0} \frac{\lambda_{kn}^2}{r_0^2} t\right] \qquad (2.10)$$

$$V_\theta = V_{\theta c} + \sum_{n=2}^{\infty} \sum_{k=1}^{\infty} \left[-\frac{A_n(x_{0k})}{n+1} x_k^{-1/2} I_\alpha(x_k) + \frac{B_n(x_{0k})}{n} x^{-1/2} I_\beta(x_k) + K_n(x_{0k}) F(x_k)\right] \frac{q_n(x_{0k})}{q_n'} \exp\left[-\frac{\mu_0}{\rho_0} \frac{\lambda_{kn}^2}{r_0^2} t\right]$$

where the subscript c denotes the pressure and velocity components in the quasistationary solution (1.7) corresponding to $S = 0$.

As the amplitude of the pressure grows, the particle strain rate becomes so great that it is impossible to consider the number Re small. In this case the deformation is similar to the spreading of a drop of ideal fluid. If the time of external-pressure action is short but the pressure amplitude is large, then the particle acquires a finite impulse during a short time. A velocity field originates instantaneously in the volume of a particle subjected to impact and is described by the hydrodynamic potential equations

$$\Delta\varphi = 0, \quad \mathbf{V} = \operatorname{grad} \varphi \qquad (2.11)$$

As the particle spreads, its shape changes and should be determined during the solution of the problem for a rigorous formulation. For a qualitative estimate, it can be considered that the particle is converted into an oblate ellipsoid of revolution with eccentricity varying with time during spreading. Let us consider the problem in an ellipsoidal coordinate system α, β, γ by taking the previous loading scheme (Fig. 1) and taking account of the impulse distribution $I(\beta)$ on the ellipsoid surface in conformity with the Newton drag law

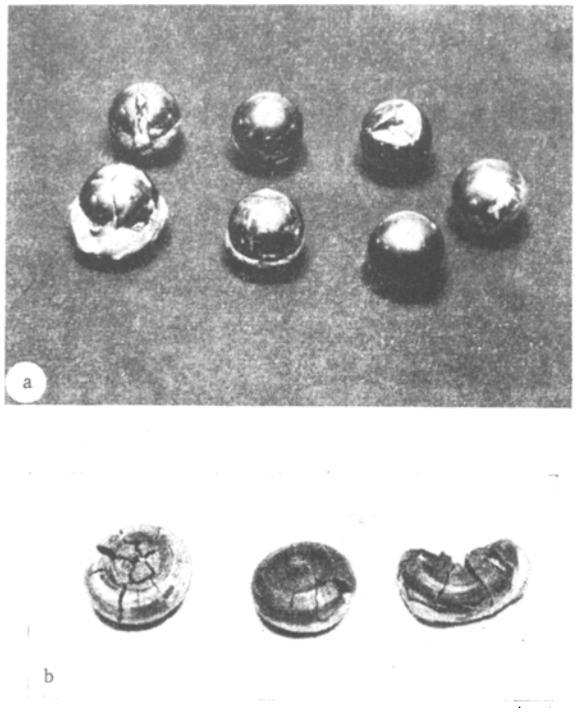

Fig. 2

$$I_\alpha = I_0 \operatorname{ch}^2 \alpha_0 \cos^2 \beta/(\operatorname{ch}^2 \alpha_0 - \sin^2 \beta), \quad 0 \leqslant \beta \leqslant \pi/2$$

The solution of the Laplace equation for the internal ellipsoidal domain with the boundary condition $\varphi|_{\alpha=\alpha_0} = -I_0 f(\beta)/\rho_0$ is

$$\varphi = -\frac{I_0}{\rho_0} \sum_{n=0}^{\infty} f_n \frac{P_n(i \operatorname{sh} \alpha)}{P_n(i \operatorname{sh} \alpha_0)} P_n(\cos \beta) \;;$$

hence, we obtain for the velocity components

$$V_\alpha = -\frac{I_0}{\rho_0 hc} \sum_{n=1}^{\infty} \frac{f_n}{P_n(i \operatorname{sh} \alpha_0)} \frac{\partial P_n(i \operatorname{sh} \alpha)}{\partial \alpha} P_n(\cos \beta)$$

$$V_\beta = -\frac{I_0}{\rho_0 hc} \sum_{n=1}^{\infty} f_n \frac{P_n(i \operatorname{sh} \alpha)}{P_n(i \operatorname{sh} \alpha_0)} \frac{\partial P_n(\cos \beta)}{\partial \beta} \qquad (2.12)$$

$$h = (\operatorname{ch}^2 \alpha - \sin^2 \beta)^{1/2}, \quad \operatorname{sh} \alpha_0 = b/c, \quad \operatorname{ch} \alpha_0 = a/c$$

where c is the focal length, and a and b are the major and minor semiaxes of the ellipsoid.

It follows from an analysis of (2.12) that the normal rate of contour deformation diminishes at $\beta = \pi/2$ as the ratio a/b grows, while the tangential deformation remains constant. If $a \to r_1$, $b \to r_1$, $c \to 0$, then the ellipsoid is converted into a sphere. The initial velocity field will here be

$$V_r = \frac{I_0}{\rho r_1} \sum_{n=1}^{\infty} n f_n \left(\frac{r}{r_1}\right)^{n-1} P_n(\cos \theta)$$

$$V_\theta = \frac{I_0}{\rho r_1} \sum_{n=1}^{\infty} f_n \left(\frac{r}{r_1}\right)^{n-1} \frac{\partial P_n(\cos \theta)}{\partial \theta} \qquad (2.13)$$

Analysis of (2.13) shows that the main mass of particle material spreads outwardly under a pulsed effect, with the exception of a rear compact zone within the sector θ = 160-200°, which is impressed in the particle under the effect of inertial forces.

The spreading of spherical particles consisting of a nucleus with density ρ_2 and radius r_2 and a shell with density ρ_1 and outer radius r_1 is investigated analogously. The solution for the potentials in the nucleus φ_2 and the shell has the form

$$\varphi_1 = \sum_{n=0}^{\infty} (A_n r^n + B_n r^{-n-1}) P_n(\cos \theta),$$

$$\varphi_2 = \sum_{n=0}^{\infty} D_n r^n P_n(\cos \theta) \qquad (2.14)$$

To determine the constants A_n, B_n and D_n, the condition of equality of the velocities $(\partial\varphi_1/\partial r)_{r=r_2} = (\partial\varphi_2/\partial r)_{r=r_2}$ and the impulses $\rho_1\varphi_1(r_2) = \rho_2\varphi_2(r_2)$ at $r = r_2$ should be used in the problem in addition to the conditions on the outer contour $\varphi_1(r_1, \theta) = -I_0 f(\theta)/\rho_1$. For $\rho_2 = 0$ (2.14) yields the solution of the problem of spreading of hollow particles.

3. Experimental Results. The experiments were conducted by using a ballistic apparatus which permitted the firing of steel balls of 9-15-mm diameter (3-13 g in weight) at velocities up to 2.5 km/sec. Quasistatic modes of particle loading were considered in the first series of tests, whereupon the particles were decelerated in aluminum blocks whose thickness exceeded the particle diameter many times. Annealed and incandescent particles, i.e., viscous and brittle, were examined. After penetration the balls were extracted from the blocks in each experiment and the mechanism of their destruction was investigated. Photographs of the incandescent balls from the rear surfaces are presented in Fig. 2a. It is seen from the photographs that discontinuities because of the tensile stress $\sigma_{\varphi\varphi}$ originating in the "equatorial" region of the ball are the main kind of destruction.

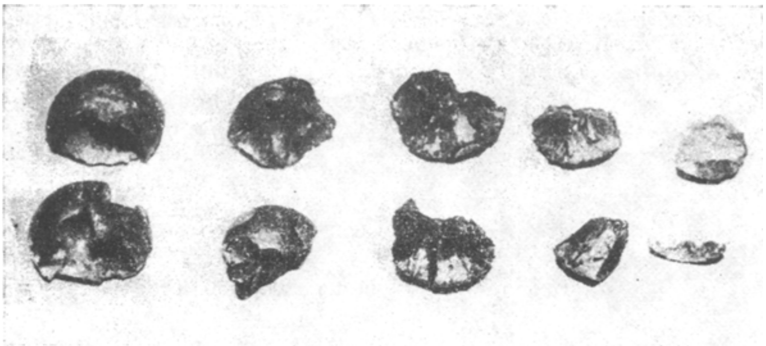

Fig. 3

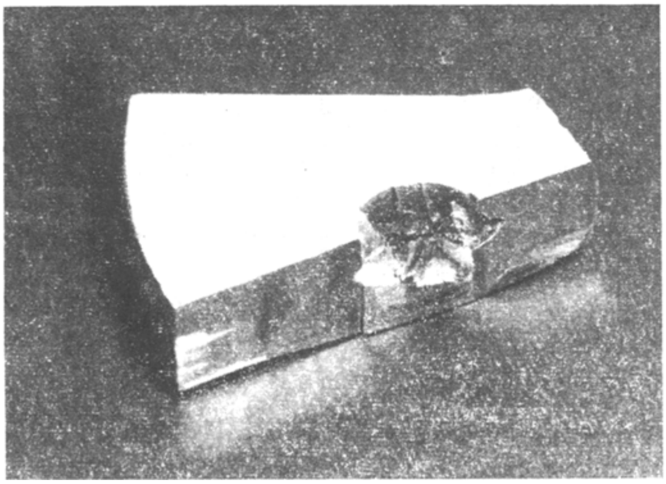

Fig. 4

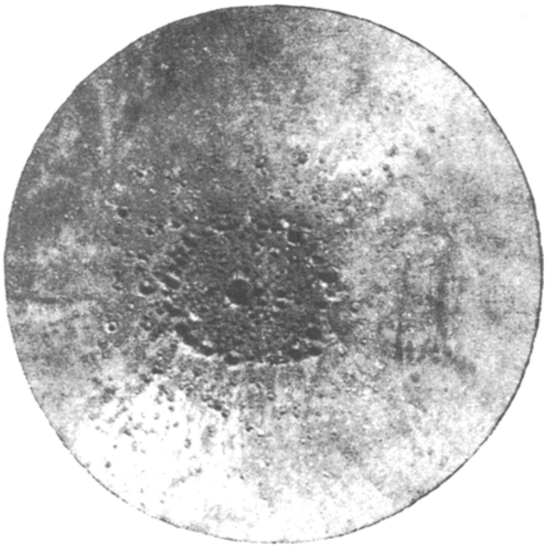

Fig. 5

Radial cracks are clearly expressed in the ellipsoidal particles (Fig. 2b), where they occupy a relatively large volume as compared with the spherical particles. The next series of experiments was conducted with annealed (viscous) particles, whose main mechanism of destruction is a shear mechanism. These experiments confirm the presence of a compact head zone, which retains its completeness during particle destruction (Fig. 3). The compact zones have the shape of cones with a $\sim 90°$ vertex angle. The rest of the spheroid material slides over the generator of this cone, being spread outward in conformity with the theory of viscous outflow and the theory of elasticity.

Presented in Fig. 4 is a photograph of a crater in a section formed in the thick aluminum obstacle for a $V = 2 \cdot 10^5$ cm/sec ball velocity. At the center of the crater there is a rise formed by the undestroyed compact head zone of the particle. The creater is similar to a lunar cirque, adding therefore to the hypothesis [12] of the possible mechanism for their formation.

To estimate the angles of dispersion of the secondary particles formed during destruction of the main particle, lead balls were used which were completely destroyed in the velocity range mentioned. Their behavior is described well by viscous and ideal fluid models. The obstacle was $h/d_0 = 0.6$ thick.

As the velocity grows, the angle of dispersion of the secondary particles increases, tending to a finite value in conformity with the result of theory. Presented in Fig. 5 is a photograph of a block located at some distance from a thin ($h/d_0 = 0.5$) obstacle punched through at a velocity $V_0 = 5$ km/sec on which is seen the "ring" dispersion of the secondary particles, due to the intensive displacement of the particle material to its peripheral regions, as follows from theory.

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